Dynamic Dictionary of Mathematical Functions (DDMF)

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DDMF Team:
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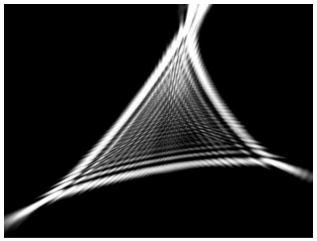
A. Darrasse, S. Gerhold (former post-docs)

Microsoft Research – INRIA Joint Centre

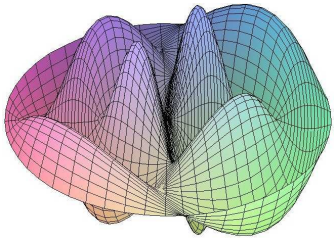
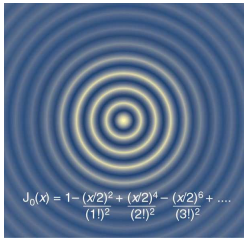
April 12, 2011

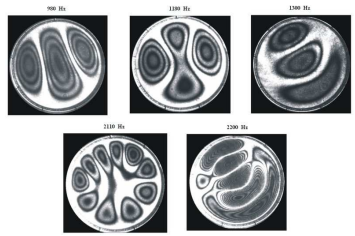
Plan

Special Functions: From Physics to Applied Mathematics

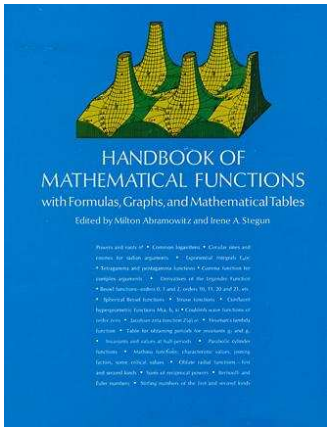
Airy function

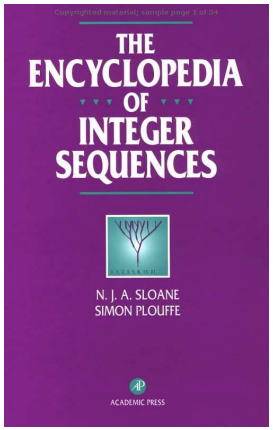



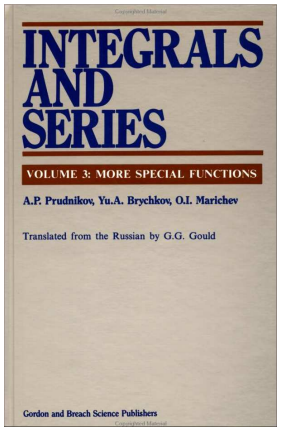


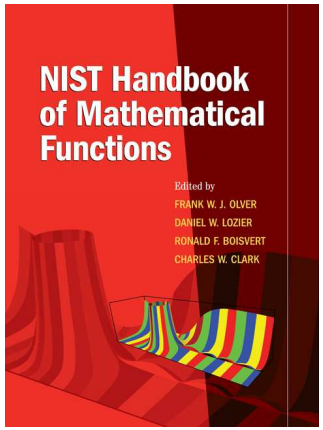
Bessel function

Own theory developed in mathematical analysis (19th and 20th centuries).









Algorithms developed in *Computer Algebra* since the 1980's.

Dynamic Dictionary of Mathematical Functions (DDMF)

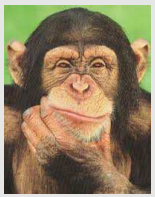
DDMF = Mathematical Handbooks + Computer Algebra + Web

Aims of the project:

- Develop a public dictionary online:

→ `http://ddmf.msr-inria.inria.fr/`
→ *DDMF release 1.6* (Nov. 2010)

- Develop new computer-algebra algorithms to generate more formulas on Special Functions:



→ linear differential equation as a data structure,
→ implementation: *Algolib*.

- Develop tools for Dynamic Mathematics on the Web:

→ interactivity + incremental computations,
→ implementation: *DynaMoW*.

Plan

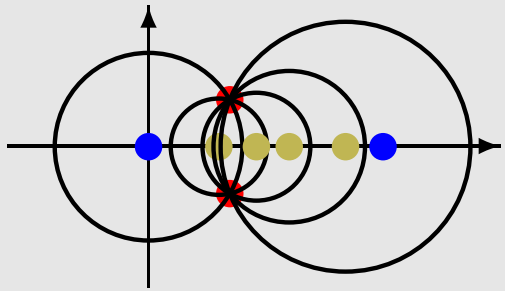
Progress of the DDMF: Numerics and Approximations

Fast, guaranteed arbitrary-precision numerics

[A. Mezzarobba, B. Salvy]

→ effective bounds

\+ analytic continuation:



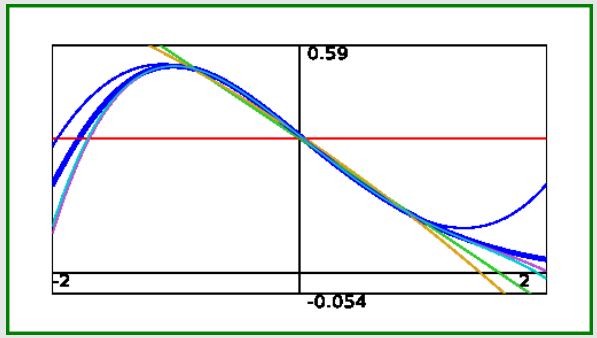
Approximation by Chebyshev series

[A. Benoit, B. Salvy]

→ non-commutative fractions

\+ fast algorithm:

$$f(x) = \sum_{n=0}^{\infty} c_n T_n(x)$$



Progress of the DDMF: Sums and Integrals

Integral transforms

[B. Salvy, S. Shankar]

$\rightarrow$ implementation for Laplace transforms: $f^*(s) = \int_0^\infty e^{-st} f(t)\, dt$

General integrals and sums

[F. Chyzak, L. Pech]

$\rightarrow$ non-commutative elimination + fast implementation:

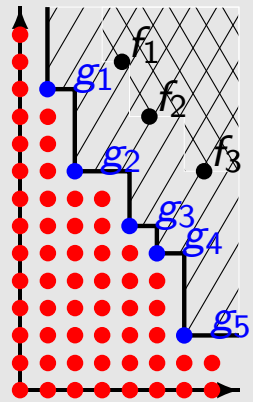
$$\int_0^{+\infty} xJ_1(ax)I_1(ax)Y_0(x)K_0(x)\, dx = -\frac{\ln(1-a^4)}{2\pi a^2}$$

Towards more special functions

[F. Chyzak, M. Kauers, B. Salvy]

$\rightarrow$ fewer independent equations than free variables:

$$\int_0^\infty x^{\alpha-1} \operatorname{Li}_n(-xy)\, dx = \frac{\pi(-\alpha)^n y^{-\alpha}}{\sin(\alpha\pi)}$$



Plan

Dynamic Mathematics on the Web (DynaMoW)

Goals

1. Interactivity and incremental computations
   → dynamic generation of documents by computer algebra
2. Automatic writing of proofs / certification of results
   → trace of the symbolic computation is part of the output

web client

$\longleftrightarrow$

DynaMoW =
Web Services
+ CAS Plugins
+ Math. Rendering

$\longleftrightarrow$

math. server

DDMF = DynaMoW + Specific Special-Functions Knowledge

DynaMoW, an Ocaml Library

[F. Chyzak, A. Darrasse]

DynaMoW = Ocaml + quotations + antiquotations

Symbolic result, converted to LaTeX, put into a paragraph

```
let res = <:symb< symbolic expression >> in
  <:par< some text <:imath< some latex $(symb:res) >> >>
```

Using Ocaml values in symbolic computations

```
let n = 23 and s = "foo" in
  <:symb< f($(int:n), $(str:s)) >>
```

Symbolic objects cast to Ocaml types

```
let n = 23 + <:int< symbolic expression >> in ...
if <:bool< symbolic expression >> then ... else ...
<:unit< f := symbolic expression >>
```

1.0.0 beta (released March 2011) + user's manual (wip)

Plan

Short DDMF Demo Now!

DDMF release 1.6 (Nov. 2010):
`http://ddmf.msr-inria.inria.fr/`

Ongoing & Future Work

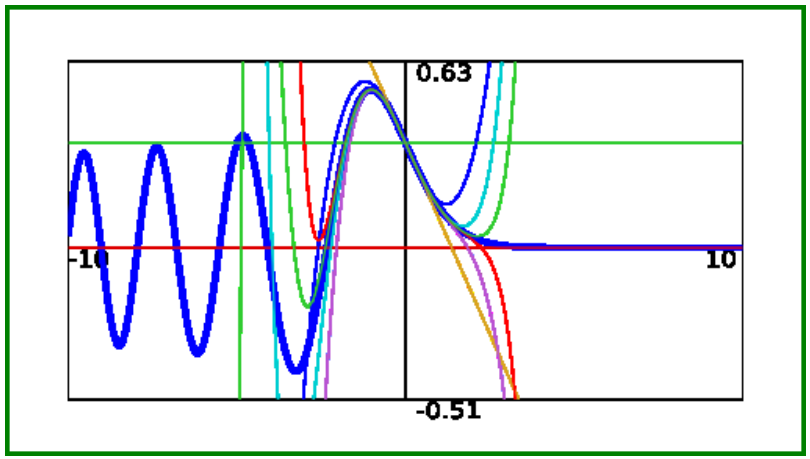
More content on the web (new people underlined)

- Certified uniform numerics [Benoit, Joldeş, Mezzarobba].
- Branch cuts [Chyzak, Davenport, Salvy].
- More explicit asymptotics [Chyzak, Stan].
- Parametrised functions [Chyzak, Koutschan].
- More integral transforms [Peasgood, Salvy].
- Plots: definition domains, automatic ranges, navigation.
- Special sequences and orthogonal polynomials.
- Sums and integrals: fast algos, multiple sums and integrals.

Extensions

- Extraction of symbolic code?
- Formal proofs?
- DynaMoW for mathematical education?

Don't miss B. Salvy's demo!



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